Lagrangian Dispersion in Turbulent Flow from Laser Transit Anemometry

The Lagrangian dispersion function in turbulent flow can be determined from measurements of the probability that a particle which is observed in a small volume is not observed in a similar size volume downstream. In this work a single-detector method is proposed to detect the particle arrivals in a multiple series of scattering volumes; the probability can be determined from the digital record over a long time compared to the turbulent integral time scale. The method is tested at locations downstream of a water spray nozzle and a submerged jet of dilute polymer solutions.

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SCOPE

The transport or dispersion of a passive contaminant by turbulent convection is a stochastic process that can be analyzed in an Eulerian or a Lagrangian frame of reference. Although the Lagrangian or random trajectory model is easiest to formulate, most measurements are Eulerian and are made at fixed points in space. One Lagrangian experiment is the measurement of the time of flight of particles between two points in the flow illuminated by a focused laser beam. When the points are close together so that the flight time is much less than the integral time scale of the flow, dispersion is not a parameter and the two frames of reference are the same. Most applications of a time of flight or laser transit anemometer are based upon the location of the two spots at such close proximity to each other.

When the separation of the spots is of the order of the integral scale, not all the particles that are observed in the first spot will be seen in the second spot. The probability of finding a particle in the first spot and not in the second spot can be experimentally measured, and this probability can be shown to be only a function of the dispersion. The theoretical development and some experiments using two photodetectors have been previously presented by Tan and Berman (1982) and Berman et al. (1984). However, this method requires careful alignment and realignment for each setting of the space between the beams.

In this paper the theoretical development of a simpler technique using a single photodetector is described and tested experimentally. The method to measure the turbulent mixing parameters in a Lagrangian framework can be valuable in studies of mixing in process equipment and in two-phase flows.

CONCLUSIONS AND SIGNIFICANCE

The probability aftereffect is the combination of the events that some particles which were originally in the first spot move out and that some particles move in from the surrounding fluid during the time of flight between the spots. In turbulent flow the probability distribution of the particle arrivals within a sampling time interval is approximately Poisson with a ratio of variance to mean close to unity. There is a relationship between the probability aftereffect and the mean square difference between the number of particles in the first spot in the time interval and the number of particles in a similar time interval but separated by the flight time. For the almost Poisson distribution this mean square difference becomes the mean number of particles that pass through only one beam. Therefore, a single-photodetector system which sees both spots can be used in place of a two-photodetector system that registers which spot the particle passes through.

Experiments were made to verify the almost Poisson distribution, the feasibility of the single-photodetector technique, and the determination of the Lagrangian parameters for the dispersion. For sampling times larger than the integral scale, the major contribution to the deviation from a Poisson distribution was found to be the dead time of the signal processing. An experimental apparatus using a calcite prism to provide the spot separation was used in a spray nozzle and the data obtained with a single photodetector gave a turbulence intensity in agreement with other measurements on a similar spray nozzle.

The multiple-spot technique and data analysis was illustrated with the previous two-photodetector system in submerged jets of dilute polymer solutions. An exponentially delaying correlation function has two parameters, the rms velocity fluctuations and the integral time scale. These parameters can be combined to give the energy dissipation and the diffusivity. At the center of the jet 60 diameters from the jet source, the dissipation was the same for the dilute polymer solutions and the pure solvent. However, the diffusivity was greatly reduced in the polymer solution jets. This experiment illustrates the usefulness of the laser transit anemometer as a measuring tool to obtain quantitative values of turbulent parameters and to obtain an insight into the dynamics of the flow and mixing.

Introduction

In a laser transit anemometer the time a particle takes to travel between two focused laser beams is measured. From many such realizations the mean velocity and turbulence intensity in the direction of the particle path between the two beams can be obtained. The beams must be placed close together to insure that there is no change in velocity along an individual particle path. Directional properties of the flow can be obtained by changing the orientation of the two beams. When the two beams are lined up in the mean flow direction, turbulent diffusion in the cross-flow direction leads to the probability that a particle will intersect only one of the beams rather than two. Measurements of this probability as a function of distance between the beams has been proposed as a method to determine the Lagrangian turbulence intensity and integral scale (Tan and Berman, 1982; Berman et al., 1984).

In this paper the experimental technique is applied to a dilute polymer solution jet flow, and a new experimental procedure is developed that uses a single photodetector to record particle arrivals. The speed of data processing and ease of alignment are greatly improved compared to the original method of Tan and Berman. This experimental technique is ideally suited to the study of turbulent mixing in process equipment and in two-phase flows. When Eulerian velocities are available for the same flow, the results can be used to find the relationship between the two frames of reference.

Theory

The relationship between the geometric parameters, the flow parameters, and the probability of finding a particle in the first beam (A) and not finding it in the second beam (B) is given by Erdman (1983) as

$$1 - P(\tau) = (\beta/V) \iint W_1 W_2 \phi \, d\mathbf{r}_1 \, d\mathbf{r}_2 \tag{1}$$

The probability, P, has been called the probability aftereffect and the parameters in Eq. 1 are the detection probability of the photodetector, β; the scattering volume, V, assuming both beams are the same; the functions describing the size of the scattering volumes, W_i; the spatial distribution function, ϕ; and the position coordinates, $\mathbf{r}_i$. The spatial distribution function can be defined and related to the Lagrangian velocity as follows:

$$\phi = [2\pi F(\tau)]^{-3/2} \exp\{-[\mathbf{r}_2 - \mathbf{r}_1 - \omega(\tau)]^2/2F(\tau)\}$$

$$\omega(\tau) = \mathbf{v}_0\tau - \mathbf{v}\gamma(\tau)$$

$$\gamma(\tau) = \int_0^\tau \rho(t)\,dt$$

$$F(\tau) = \overline{X^2} = 2\sigma^2 \int_0^\tau (\tau - t)\rho(t)\,dt$$

where $F(\tau)$ is the same as the mean square displacement $\overline{X^2}$, $\rho(t)$ is the Lagrangian correlation function, σ^2 is the mean square velocity fluctuation, $\gamma(\tau)$ is the correlation integral, $\mathbf{v}_0$ is the mean velocity, and $\mathbf{v}$ is the fluctuating velocity. These equations represent a stochastic path integral along the path of the scattering particle. Since $\mathbf{v}$ and $\mathbf{r}_1$ are related, it is convenient to integrate over $d\mathbf{v}$ instead of $d\mathbf{r}_1$. For a Gaussian velocity distribution the integration for homogeneous, isotropic, stationary turbulent flow with long thin parallelepiped scattering volumes is given by Berman et al. (1984). When

$$\rho(t) = \exp(-t/T_0)$$

where T_0 is the Lagrangian integral time scale, the probability P becomes a function of σ^2, T_0, τ, and d/L. The only geometric scales are the scattering volume diameter or side, d, and the length, L. These are assumed to be fixed at the $1/e^2$ intensity points of the laser beams so that the W_i are equal to one and the limits of the integrals are defined by these distances. The average time of flight between the two spots is equal to ℓ/v_0 where ℓ is the distance between the two spots. The result is

$$\begin{aligned} 1 - P &= (1/2)[1 + H_1(\alpha_1)][H_2(\alpha_2)][H_3(\alpha_3)] \\ H_i &= \{[\exp(-\alpha_i^2) - 1]/\alpha_i\sqrt{\pi}\} + \operatorname{erf}(\alpha_i) \\ \alpha_1 &= \alpha_2 = d/\sigma\gamma(\tau)\sqrt{2}, \ \alpha_3 = L/\sigma\gamma(\tau)\sqrt{2} \end{aligned} \tag{2}$$

The probability, P, can be obtained from the particle arrival records in the two spots. When Chandrasekhar's (1943) derivation for Brownian motion is applied to the flow problem, the result is

$$P = \left\{ \sum_i (M_i - N_i)^2 \right\} \Big/ \left[\sum_i M_i + \sum_i N_i \right] \quad (3)$$

where M_i is the number of particles that pass through the first spot in time period T_s, and N_i is the number of particles that pass through the second spot in a time period of the same duration but displaced by the average time of flight. The derivation of Eq. 3 assumes that the particle arrivals have a Poisson distribution and the particles are distributed uniformly in space. However, particle arrivals in turbulent flow do not have a Poisson distribution (Erdman, 1982; Hirleman et al., 1984). It is possible to rederive Eq. 3 to account for the effect of turbulence if we assume that the distribution of particle arrivals has only a small deviation from a Poisson distribution.

In order to examine the statistics of the particle arrivals it is convenient to consider a conveyer belt moving between volumes A and B. For a sampling time T_s the particles are tracked in a rectangular parallelepiped initially bounded by the cross section S of A and extending upstream a length $v_0 T_s$. This volume is viewed only at A and B and we are interested in the number of particles seen in A and then in B. When the frequency distribution is almost Poisson, we can let the ratio of the variance of M, $VAR\ M$, to the mean M, $\langle M \rangle$, be δ, which is near unity. From Erdman (1982), the first-order approximation to δ is

$$\delta = 1 + c^2 S^2 \sigma^2 \gamma^2 / \langle M \rangle \quad (4)$$

where c is the particle concentration. If we substitute $cS = \langle M \rangle / v_0 T_s$ and $\gamma = T_0[1 - \exp(-T_s/T_0)]$, we see that δ is close to unity if $T_s \gg T_0$.

During the flight time τ between A and B some y particles enter the volume and some x particles remain. The probability distribution of x is Bernoulli (Chandrasekhar, 1943) with $\langle x \rangle = n(1 - P)$ and $VAR\ x = nP(1 - P)$. The particle arrivals y over the time period τ have $\langle y \rangle = \langle M \rangle P$ and $VAR\ y = \langle M \rangle P\delta$. Here the correlation integral in δ is $T_0[1 - \exp(-\tau/T_0)]$, but the substitution for cS retains the dependence upon T_s.

The mean and variance for the number of particles N seen in B over time period T_s is found from the sums of the means and variances of x and y to give

$$\langle N \rangle = M(1 - P) + \langle M \rangle P$$
$$VAR\ N = MP(1 - P) + \langle M \rangle P\delta.$$

From the ensemble average of particle counts in A and B for many time periods all of the same duration, the probability P can be determined. Let $\Delta_M = N - M$ and average over N

$$\Delta \equiv \langle \Delta_M \rangle = \langle N \rangle - M = (\langle M \rangle - M)P$$
$$\Delta^2 \equiv \langle \Delta_M^2 \rangle = \langle (N - \langle N \rangle)^2 \rangle + (\langle N \rangle - M)^2$$
$$= P^2[(\langle M \rangle - M)^2 - M] + (M + \langle M \rangle \delta)P$$

Then averaging over M we find

$$\langle \Delta \rangle = 0 \text{ or } \langle M \rangle = \langle N \rangle$$

and

$$(\delta - 1)\langle M \rangle P^2 + (\delta + 1)\langle M \rangle P = \langle \Delta^2 \rangle$$

For δ close to unity

$$P = (2/(1 + \delta))\{\langle \Delta^2 \rangle / 2\langle M \rangle\} \quad (5)$$

and the correction factor to Eq. 3 is $2/(1 + \delta)$.

The experimental technique based upon Eq. 5 requires that the particle records in the two beams be separately identified. If it is possible to identify the particles which went through two spots, say a, and those particles which only passed through one spot, say b, then a single record can be used.

Let $M = a + b_M$ and $N = a + b_N$. Then

$$P = [2/(1 + \delta)]\ \langle (b_M - b_N)^2 \rangle / [\langle M \rangle + \langle N \rangle]$$
$$= [2\delta/(1 + \delta)][\langle b_M \rangle + \langle b_N \rangle] / [\langle M \rangle + \langle N \rangle]$$
$$= [2\delta/(1 + \delta)]b/[2a + b] \quad (6)$$

For sufficiently long records the correction factor can be neglected.

Both Eq. 5 and Eq. 6 assume that the detector is perfect, but when one particle is in the scattering volume, it is usually impossible to detect a second particle. These dead time errors have been well studied for Geiger counters and several examples are given by Feller (1968). If the turbulent correction and the dead time error correction are independent, the first-order correction to be included in δ would be of the form $1 - \text{const}\ \langle M \rangle \lambda / T_s$. When the dead time λ is the time for a particle to pass through the volume and the detector is locked during this time, the constant is positive. For the single-detector system, the dead time is at least a multiple of the flight time, and a separate particle arrival would not be counted. Other digital detection schemes can result in a negative constant. In any case the error is small only if $\lambda \langle M \rangle \ll T_s$.

Experimental

The method used previously, based upon Eq. 5, requires two photodetectors and a method to change the beam spacing. In the experiments reported herein for a dilute polymer solution submerged jet this meant realigning the optics for each setting of the distance between spots, and analyzing the data after the probability at all distances had been measured. An experimental apparatus to determine P based on Eq. 6 and using the properties of a calcite prism as suggested by Hirleman (1982) for a time-of-flight laser velocimeter is shown in Figure 1. The laser beam is focused through calcite prisms so that any number of equal-intensity spots at known distances from each other can be created. Light scattered from particles passing through these spots is imaged on a single photodetector. The output of the photodetector is then processed to determine the record of particle arrivals and the dispersion. When the particle density is low, closely spaced particle arrivals are attributed to motions through more than one beam or the a in Eq. 6. The time of flight and the cutoff time for determining a are found from a probability analysis of the record or from any other laser transit anemometer technique.

To illustrate the new technique an FM tape was prepared

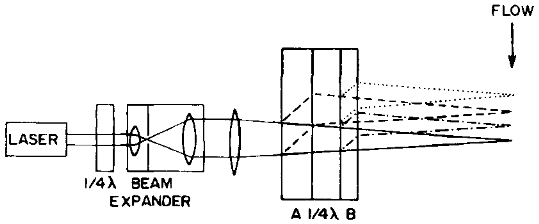

Figure 1. Diagram of multispot laser transit anemometer.

Two calcite prisms, *A* and *B*, produce four equally spaced spots. Any number of one-quarter-wave plates (to circularly polarize the beam) and calcite prisms (to split the beam) can be combined to increase the number of spots.

from the photodetector output of a two-spot laser transit anemometer similar to that shown in Figure 1. The laser beam from a 5 mw helium-neon laser was separated into two beams that were focused 0.5 mm apart along the centerline of a jet spray nozzle approximately 20 diameters from the jet source. The beams had waists of 0.36 mm and lengths of 2.15 mm to the $1/e^2$ intensity points.

The tape was analyzed by playing back at one-eighth speed. The tape recorder output was sent to the input of Hewlett Packard 5328A Universal Counter on which the threshold level was set so that only the highest voltages were detected. Then the marker output was used to supply a constant voltage level to an analog-to-digital converter. The digital record contained two levels, low values when no particle was in the volume and high levels when a particle was in the volume.

In order to find the diffusivity it is necessary to vary the distance between spots. More than two spots have not been used along with calcite prisms, but the technique will be illustrated with the experimental equipment described previously (Tan and Berman, 1982). The submerged jet and polymer solutions were the subject of an extensive laser Doppler velocity study and are described by Berman and Tan (1985). In this work measurements were made 60 diameters from the exit of the 2 mm dia. jet. The spot size was 0.06 mm waist and 0.6 mm length. The polymer solutions were 100 ppm of Polyox WSR 301 (PEO), a poly(ethylene oxide) of molecular weight about 2×10^6, and 100 ppm of Betz 1120 (PAM), a partially hydrolyzed polyacrylamide with molecular weight about 10^7. Both were dissolved in water that was deionized and then passed through a Barnstead Nanopure deionizer.

The signal analysis technique was the same as for the single-record method except that two channels of the universal counter and two channels of the analog-to-digital converter were used. The digitizing rate for the water or solvent data was 20,000/s and for the polymer solutions, 8,000/s.

Results and Discussion

Single-photodetector method

The distribution of waiting times in the digital record of the particle arrivals for the spray nozzle had a peak corresponding to a mean velocity of 2 m/s. There were 59 pairs of particle events within two standard deviations of this peak, so $a = 59$. There were also 59 more widely spaced individual particle detections, so $b = 59$. Therefore, $P = 1/3$. If we assume that the average time of flight of 0.25 ms is much less than the integral scale, the correlation integral is just the flight time and the root mean square velocity fluctuation is the only unknown in Eq. 2. Then this root mean square velocity fluctuation is found to be 0.4 m/s or 20% of the mean velocity.

Axial velocity measurements on a similar spray nozzle have been made by Bachalo and Houser (1984). Larger droplets spray outward from the axis of the jet, leaving small particles in the jet center which follow the flow of the air in the jet. At a similar location in the jet, Bachalo and Houser found particle diameters from 1 to 10 μm in diameter. The rms velocity fluctuations were about 20% of the mean velocity for the smaller particles and more for the larger particles. Many of the larger particles retain the higher velocities due to their inertia. The threshold detection used in this work is more consistent with larger particles. Also, the integral scale can be scaled up from the previous jet results and should be about 0.5 ms. When the correlation integral is adjusted for an integral scale of 0.5 ms, the rms velocity fluctuation becomes 25%. This is representative of the 10 μ particles in the center of the nozzle. The analysis considered 20 records of 8,000 points each. This represented only 1.1 s of real time. Since this amount of time is very much greater than the integral scale, the results are statistically significant.

Two-photodetector experiments

In these experiments the probability aftereffect, P, is obtained from measurements of the number of particles observed in two volumes each imaged on a separate photomultiplier tube. The mean numbers of particles and the ratio of variance to mean, δ, were calculated at the same time. In this work $\alpha_3 = 10\alpha_2$ so the only unknown in Eq. 2 is α_2. Then

$$1/\alpha_2 = (\sqrt{2}\,\sigma T_0/d)[1 - \exp(-\tau/T_0)] \quad (7)$$

and T_0 and σ can be found from a fit to the data. Figure 2 shows the results at the center of the submerged jet and Table 1 gives a comparison of the rms velocity fluctuations and integral scales from this Lagrangian method and the laser Doppler measurements, which are Eulerian.

One of the set of measurements taken by Tan can be used to investigate the dead-time error. The equipment was set up in the evening and allowed to run overnight. During this time the photomultiplier sensitivity decreases so that $\langle M \rangle$ decreased slowly

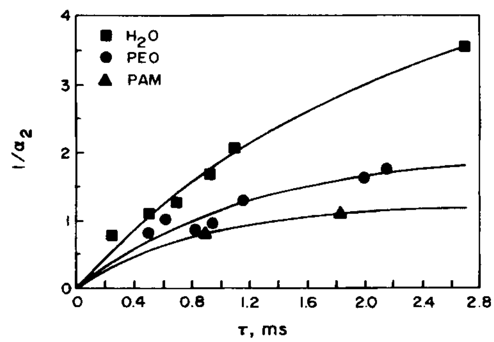

Figure 2. Correlation integral analysis of the data at the center of a submerged jet.

□ water; O 100 ppm PEO; Δ 100 ppm PAM. Solid lines are Eq. 7 with parameters given in Table 1.

Table 1. Comparison of Velocity and Time Scales

Location y/x	Fluid	Lagrangian σ, mm/s	Lagrangian T_0	Eulerian σ, mm/s	Eulerian T_E
0	water	100	2.1	90	9
0	PEO	73	1.2	90	11
0	PAM	62	0.86	65	4
0.05	water	70	3.0	65	11
0.05	PAM	50	0.8	50	5
0.1	water	50	5.0	30	20
0.1	PAM	20	2.0	16	15

throughout the night. All other parameters were held constant and the averaging time period, T_s, was set at a low value of 10 ms. Figure 3 shows δ and P as functions of the average particle number. Each point represents an average of 5,000 time periods or 50 s. The least-squares fit to these data for δ is shown in the figure and

$$\delta = 1.0 - (1.0 \times 10^{-3} \langle M \rangle / T_s) \quad (8)$$

The second constant in Eq. 8 is a constant times the dead time. An estimate of the dead time can be found by adding twice the time it takes a particle to travel through the volume plus twice the cycle time of the analog-to-digital converter. In this experiment the transit time was 0.13 ms, the cycle time was 0.1 ms, and the dead time is approximately 0.5 ms. Thus the constant which multiplies the dead time is about two.

Figure 3 also shows the corrected probability $P_0(1 + \delta)/2$ where P_0 is 0.743, the best fit to the data. The data for P, however, contain errors due to the change in alignment overnight as a result of room temperature variations. This gives a trend to higher P as the particle average decreased. For these data the turbulent contribution to δ is negligible. The data points in Figure 2 were obtained from experiments using larger averaging times, and periodic alignment checks were made to avoid errors. Application of the correction to account for dead-time errors did not lead to significant changes.

Some measurements were also made at y/x of 0.5 and 0.10. At these locations the flow is less isotropic than at $y/x = 0$, and especially at the largest radial distance from the jet center the results are difficult to interpret. A summary of the best-fit

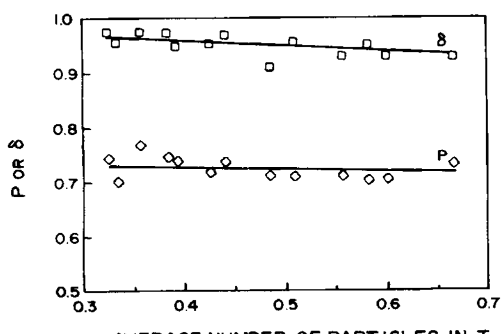

Figure 3. Experimental verification of the effect of dead time, using submerged water jet and 0.93 ms flight time.

parameters σ and T_0 is given in Table 1. Only two data points were available for PAM at each location as each point required a lengthy repeat of the mixing and refilling procedure in the once-through system. The water data have been previously reported by Berman et al. (1984).

At $y/x = 0.05$ the measured δ is less than one for both fluids and the dead time contribution is similar to that for the measurements at the jet center. However, at $y/x = 0.1$, δ is greater than one for the solvent and less than one for PAM. The rms velocity fluctuation for the solvent is 50% greater than the LDA measurements in the radial direction. The turbulent contribution does not account for the increase in δ, but the axial velocity fluctuations are much larger than the radial velocity fluctuations so the increase in σ is reasonable.

The exponentially decaying model for the correlation function can give information about the dynamics of the turbulence. Tennekes (1979) has shown that the dissipation per unit mass, ϵ, is proportional to σ^2/T_0. Table 2 gives this ratio and the dispersion at long times or diffusivity $\sigma^2 T_0$. In a jet of Newtonian fluid, the dissipation and dispersion do not change much over the central portion of the jet $y/x < 0.1$ (Hinze, 1975). The data of this work for water show some decrease in both as y/x increases but this is not unreasonable at the relatively low Reynolds number of 10,000.

Of more interest is a comparison of the solvent and the polymer solutions. At the center of the jet the dissipation is the same for all the fluids, but the dispersion is lower for the polymer solutions. The eddy diffusivity for PAM is about the same as the kinematic viscosity, indicating that there is little real turbulence in this case. For these experiments the exit velocity of the jet was the same for all fluids, and therefore the convection terms in the energy balance would be the same. The energy balance in a jet is of the form convection plus production equals dissipation plus diffusion (if pressure effects can be neglected). When the convection and dissipation remain the same in the center of the jet, the decrease in diffusion means a decrease in turbulent production. The jet is different from other flows such as pipe flows where these polymer solutions are drag-reducing and dissipation and turbulent production are balanced.

There are no changes in the relationship between the solvent and the polymer solutions at $y/x = 0.5$, but at $y/x = 0.1$ the dissipation and dispersion for PAM both drop off very rapidly compared to the solvent. The LDA studies of Berman and Tan (1985) showed that the spread of the polymer solution jets was much less than the spread of the solvent jet and the probability results show the same.

Table 2. Comparison of Diffusivity and Dissipation

Location y/x	Fluid	$\sigma^2 T_0$ $m^2/s \times 10^5$	σ^2/T_0 m^2/s^3
0	water	2.1	4.8
0	PEO	0.6	4.4
0	PAM	0.3	4.5
0.05	water	2	2
0.05	PAM	0.2	3
0.1	water	1	1
0.1	PAM	0.08	0.2

Because the jet flow is not homogeneous, the time scale preserves some memory of the upstream conditions. The measured time scale is too short for this nonhomogeneity to have an effect for a Newtonian fluid. However, for the dilute polymer solutions the time scale of the individual molecules is of the order 1 ms, and the time scales of the preferred oscillations at the origin of the jet and the flow in the constriction leading to the jet are all similar. The measured integral time scale is also the same, but the time scale of a 100 ppm polymer solution after the jet exit may be much longer than that of an individual molecule. The particle counting technique would not distinguish between turbulence and a slowly decaying oscillation in the flow preserved by the relatively long memory of interacting elongated polymer molecules. Flow visualization studies by Berman and Tan (1985) do indicate that there are nonturbulent oscillations in the dilute polymer solution jets that make Eulerian measurements of the turbulent intensity and integral time scale high. The Lagrangian analysis is consistent with these decaying oscillations when the Lagrangian time scales are characteristic of the oscillations that originate upstream of the measuring location. The Eulerian measurements on the other hand add the turbulence and the nonturbulent oscillations.

Conclusions

The Lagrangian dispersion function in turbulent flow can be determined from measurements of the probability that a particle which is observed in a small volume is not observed in a similar small volume located downstream. Detection of these particles with a single photodetector has been shown to be feasible, and the probability can be measured with only small errors when the sampling time is long compared to the flight time. When a series of scattering volumes is spaced in the mean flow direction so the time of flight spans the integral time scale of the flow, the dispersion function can be calculated at the same time as the other digital data processing.

In the example presented in this work of dilute polymer solution jets, the results of Lagrangian measurements of the turbulent intensity and integral time scale supplement the previously obtained Eulerian measurements. These results can be interpreted in terms of the dissipation per unit mass and the eddy diffusivity at long times, two important quantities that are much more difficult to obtain from Eulerian measurements. This example suggests that the laser transit anemometer would be a powerful addition to the techniques available for the study of turbulent mixing.

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Notation

a = number of particles that go through both volumes
b = number of particles that go through only one volume
b_M = particles that go through the first volume
b_N = particles that go through the second volume
c = concentration of particles
d = beam waist diameter at $1/e^2$ intensity points
F = mean square dispersion function
H_i = functions defined by Eq. 2
ℓ = distance between the center of the two spots along the mean flight path
L = beam length at $1/e^2$ points
M or M_i = number of particles in time period T_s for first spot
$\langle M \rangle$ = mean M averaged over many time periods
N or N_i = number of particles in T_s for second spot
$\langle N \rangle$ = mean N averaged over many time periods
P = probability of finding a particle in first spot and not in the second spot
r_i = coordinate vectors
S = cross-sectional area of scattering volume normal to the mean velocity vector
t = time
T_E = Eulerian integral time scale
T_0 = Lagrangian integral time scale
T_s = sample time
v = fluctuating velocity vector
v_0 = mean velocity vector
W_1 = functions describing the scattering volume
x = particles that do not leave the scattering volume during the flight time or the axial distance from the jet exit
y = particles that enter the scattering volume during the flight time or the radial distance from the jet centerline

Greek letters

α_i = functions defined by Eq. 2
β = detection probability of photodetector and instrumentation
γ = correlation integral
δ = ratio of variance to mean referring to the number of particles in the sampling time period
Δ_M = $N - M$, the difference when the time period for N is separated by the mean flight time from the time period for M
Δ = $\langle \Delta_M \rangle$ where the average is over N, holding M constant
$\langle \Delta \rangle$ = average over M
Δ_M^2 = $(N - M)^2$
Δ^2 = $\langle \Delta_m^2 \rangle$ where the average is over N, holding M constant
$\langle \Delta^2 \rangle$ = average over M
ϵ = dissipation per unit mass
λ = dead time during which the processor is not operative and a particle will not be detected, or wavelength of light
ρ = Lagrangian correlation function
σ = root mean square velocity fluctuation
τ = time of flight between spots along the mean velocity
ϕ = spatial distribution function
ω = the average distance traveled in time τ

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